EXPERIMENTAL INVESTIGATION OF LIQUID PROPELLANT DYNAMICS IN A DOUBLE CYLINDRICAL TANK

by

Guido E. Ransleben, Jr.

FINAL REPORT
Contract No. NAS8-28086
Control No. 1-2-75-20063 (1F)
SwRI Project No. 02-3363

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

May 1973

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Approved:

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Structural Dynamics and Acoustics

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LIST OF SYMBOLS

Symbol	Definition
f_N	Natural frequency of Nth mode, Hz
F_x, F_y, F_z	Forces along respective axes
g	Acceleration due to gravity
h	Height of liquid in tank
M_x, M_y, M_z	Moments about respective axes
r_o	Radius of individual tank segment
x_o	Excitation amplitude along x-axis
y_o	Excitation amplitude along y-axis
α, β	Tank tilt angles - directions shown in Figure 1.
λ	$= \log A_0/A_1 =$ logarithmic decrement of decaying slosh forces or moments, where A_0 and A_1 are amplitudes at successive cycles
Ω_o	Excitation amplitude about z-axis
ω_N	Natural frequency, radians per second

I. INTRODUCTION

Propellant dynamics in axisymmetric tanks accelerated parallel to their axes have been studied extensively, since nearly all previous rocket vehicles of interest have had axisymmetric tanks, and thrust vectors parallel to the tank axis. Abramson(1)* summarized many of these studies in a monograph on liquid sloshing. The space shuttle now being developed not only contains propellant tanks of unusual shapes to fit in an aircraft-type structural envelope, but must also be controllable in horizontal flight and during the transition from vertical to horizontal flight. Thus, the tanks for the space shuttle vehicle will not all be axisymmetric, nor will the thrust vector always be parallel to the axes of any of the tanks.

Recent work has been devoted to the study of sloshing in tilted cylindrical and other shaped tanks. Bugg(2) and Thornton and Bugg(3) experimentally determined the natural frequencies as a function of tilt and liquid depth, and McNeill and Lamb(4) and Moiseev and Petrov(5) determined the slosh frequencies analytically for cases where the free surface did not intersect the tank bottom. Chu(6) solved a similar problem for a two-dimensional tank, by numerical methods. In none of these studies were the slosh forces or moments evaluated, nor was an equivalent mechanical model, for use in control or loads analyses, formulated. In a subsequent program, Dodge and Garza(7) obtained experimental measurements of the frequencies, forces and moments, and damping, for sloshing in a tilted cylindrical tank, as a function of tilt angle and liquid depth. Also, existing analyses were extended so that an equivalent mechanical model could be developed and verified by the tests. In the cylindrical tank of that program, the experimental results indicated that the natural frequencies decreased as the tilt increased, and that the slosh damping decreased markedly with increased tilt when excited in the tilt direction. A mathematical model derived from theory confirmed the experimental observations and also indicated creation of an oscillating force parallel to the tank axis, which could be of importance in POGO applications.

In addition to this previous work, a computer program is being prepared by NASA, using a finite volume element representation of the liquid, which will compute the propellant oscillation modes and frequencies for tanks of any shape, fill level, or orientation. The purpose of the present program, reported herein, is to provide experimental data with which to verify and supplement the computed results so that the computer program may be used with confidence.

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*Superscript numbers in parentheses refer to references on page 36.

II. EXPERIMENTAL APPARATUS

A. Tank

The tank was fabricated from clear acrylic tubing and formed acrylic hemispherical sections. Tubing with a nominal 304.8 mm (12-inch) O.D., 295.3 mm (11-5/8-inch) I.D. was used for the straight upper portion. The sides were milled from two sections of tubing, and the sections bonded together with solvent. The bottom was made from two formed hemispherical sections bonded to the cylindrical sections before milling. Discontinuities at the joint between the cylindrical and hemispherical parts were filled with a mixture composed of chips of the acrylic dissolved in solvent, to smooth and fair the joint. The joint between the two halves of the tank was reinforced on the outside with epoxy.

An aluminum block containing a spherical bearing was bonded to the tank bottom with epoxy, located so the centerline of the bearing was in line with the inside bottom of the hemispherical tank bottoms. This placed the pivot point on the origin of all axes. The spherical bearing allowed tilting the tank in any direction from the vertical. An aluminum sleeve was bonded near the upper edge of the tank to provide support. Holes drilled and tapped into the sleeve mated with holes in the support braces (described later) to position the tank either vertical or tilted 10° in either the X or Y directions. Figure 1 shows the tank and its related coordinate system.

B. Support and Excitation System

The tank was mounted on a support platform which, in turn, was supported on a dynamometer system so that all loads acting on the tank were transmitted into the dynamometers, as shown schematically in Figure 2. Figure 2 also shows how the components of the forces and moments were transmitted into each dynamometer arm. The support platform was an aluminum weldment, made as rigid as possible to eliminate distortions in the load measurements due to platform deflections. Braces mounted on the platform held the tank fixed in either the vertical or tilted positions, depending on which set of screw holes was used to anchor the sleeve at the upper edge of the tank. The dynamometers were mounted on a rigid turntable which could be rotated through a large diameter bearing attached to the exciter platform. An overall view of the entire apparatus is shown in Figure 3.

The exciter platform, driven by a variable speed motor and adjustable eccentric, could be oscillated in translation along one axis only.

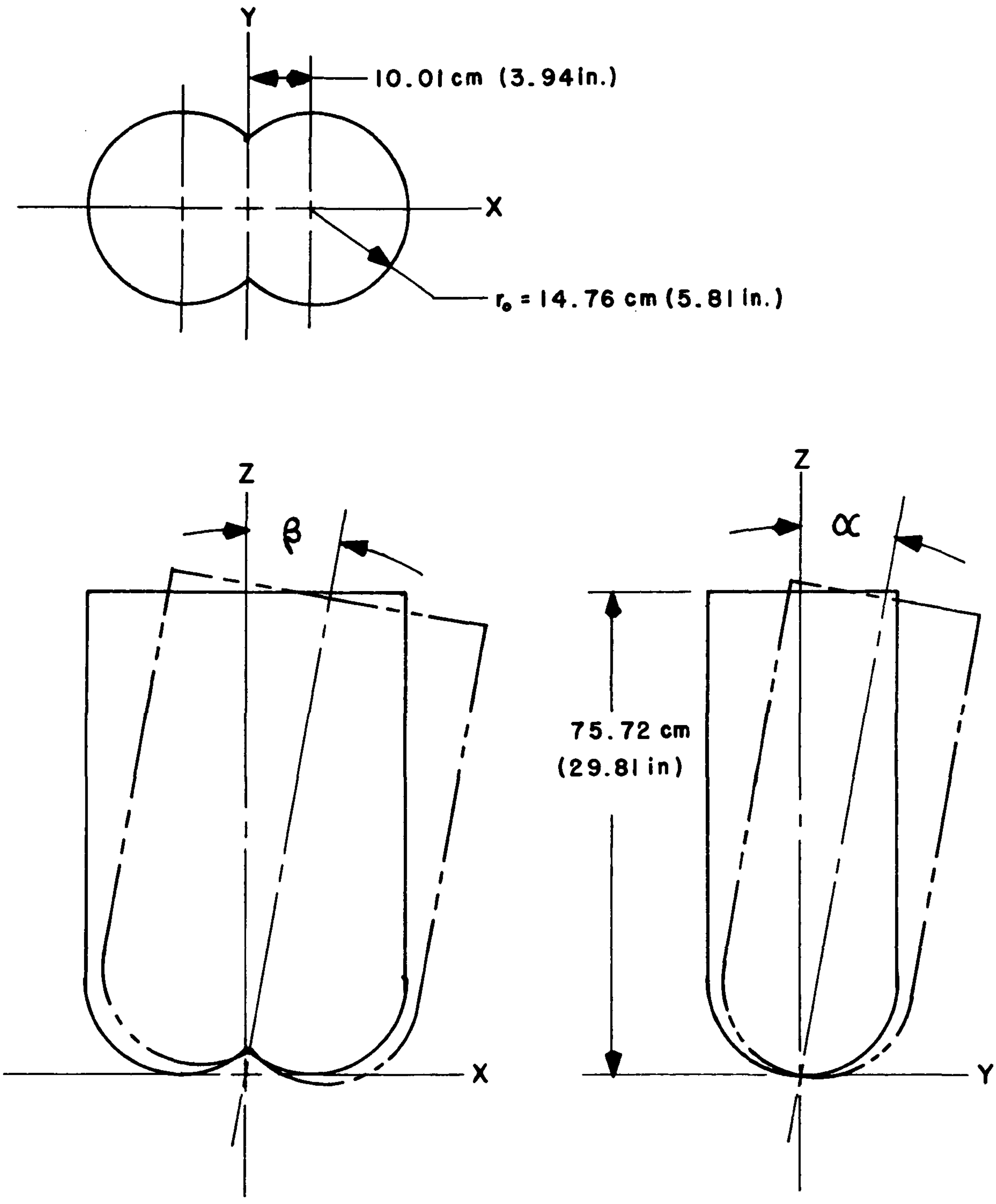


FIGURE 1. COORDINATE SYSTEM FOR TANK

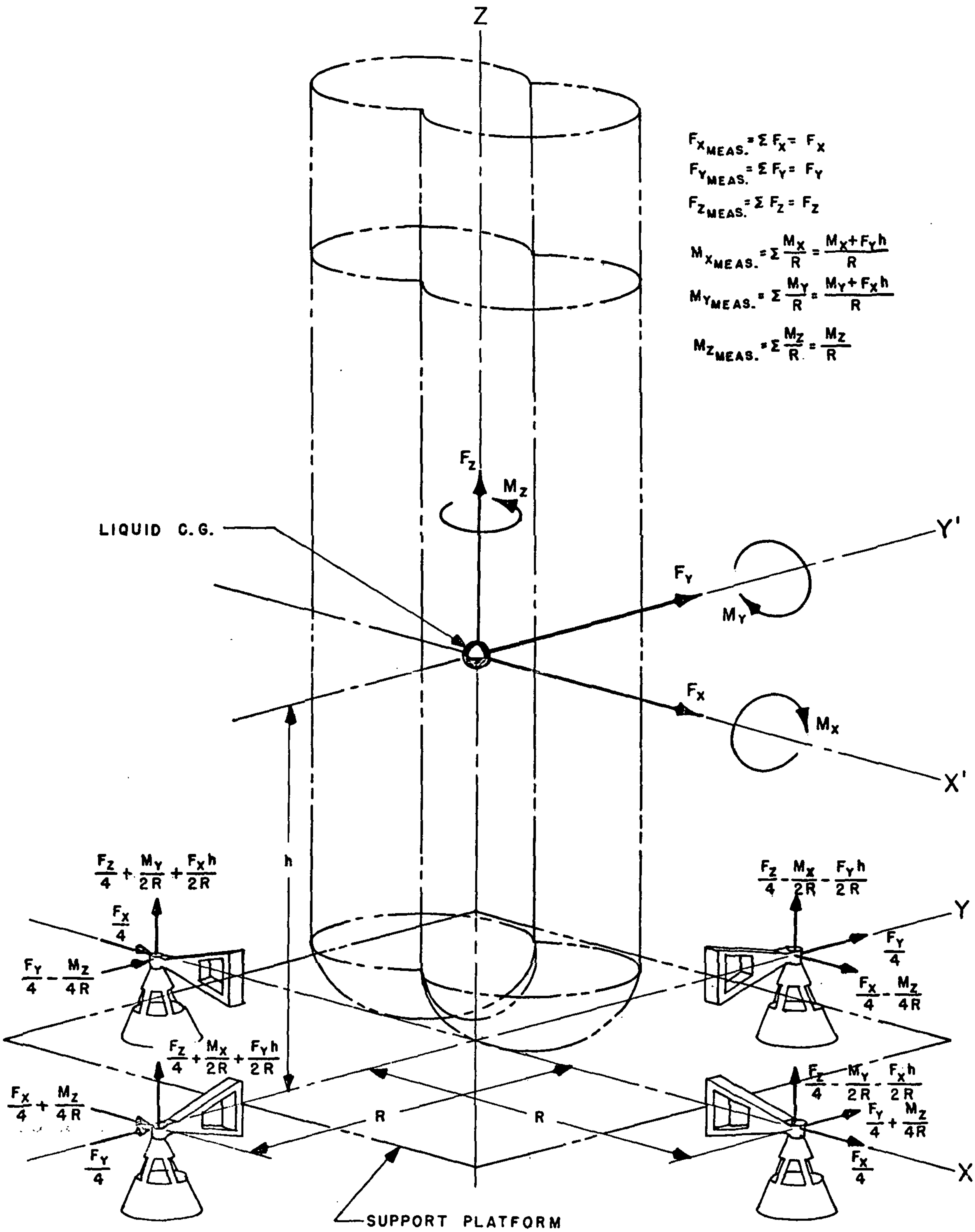


FIGURE 2. FORCE AND MOMENT DYNAMOMETER SYSTEM

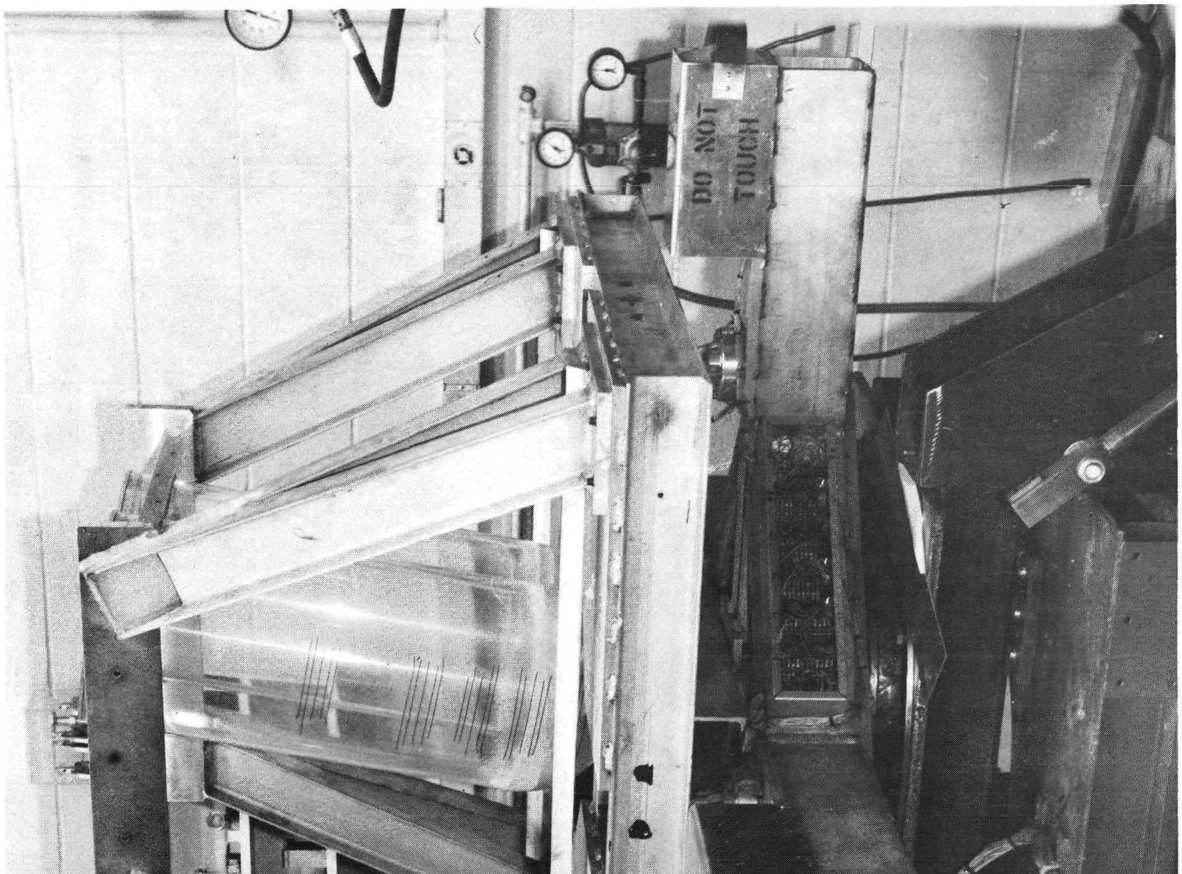


FIGURE 3. VIEW OF TEST FACILITY

The turntable could be swung 90° to allow translation along either the X or Y axis. The connecting rod between the eccentric and the exciter platform was attached to the platform through a shear connection. At one end of the stroke, a device to lock the exciter platform could be actuated (at zero platform velocity), permitting decay records of the liquid loads to be made.

The exciter system used for these tests actually has two platforms located opposite to one another relative to the drive shaft, each one being connected to separate adjustable eccentrics on the shaft. For the rotational tests (excitation about the Z-axis), the exciter platform on which the tank was mounted, was disconnected from the drive shaft and locked in position. A rod was connected between the other platform and the turntable, away from the centerline, so that lateral translation of the second platform induced rotation of the turntable on its bearing, about the tank Z-axis.

C. Instrumentation

As shown in Figure 2, the dynamometers are all triangular beam types, so loads acting normal to the beam are transmitted as essentially tension or compression of the gage elements. The horizontal arms were bolted to the tank platform, and were sensitive only to vertical forces. Summing the output of these arms yielded the total vertical force, F_z. Subtracting the output from the two arms on the X-axis yielded the moment about the Y-axis, M_y. Subtracting the output from the two arms on the Y-axis yielded the moment about the X-axis, M_x. These arms were supported on the conical, four-element arms bolted to the turntable. These arms were sensitive only to horizontal forces, and the outputs could be similarly summed in such a way as to yield the remaining three loads, F_x, F_y, and M_z. Actually, each gage element on the arms had separate sets of strain gages applied to them for each force or moment channel. Each channel was therefore complete and independent in itself so that all three forces and three moments could be recorded simultaneously.

A system of balance arms with masses attached to the ends of cantilever elements, and mounted on the turntable, was used to provide cancellation of the inertia loads due to the mass of the empty tank and tank platform. The gages of these arms were wired into the dynamometer bridges so the signals from them would cancel the dynamometer signals due to inertia. Complete cancellation of the inertia signals was never achieved, however, as the mass required to completely balance the inertia resulted in too low a natural frequency of the balance arms. The arms were therefore ballasted only until their natural frequency was on the order of 16 Hz — approximately four times the highest anticipated excitation frequency. Before testing in each configuration of tilt or excitation direction, a series of runs with the empty tank was made at several frequencies covering the test frequency range, so a plot of the residual inertia signals versus frequency could be made. These plots were then used during the data reduction to subtract the residual inertia from the recorded forces and moments. No balance arms were required in the F_z bridge, as there was no excitation (and no inertia) in the Z-direction with the tank vertical, and only a small component with the tank tilted.

A very flexible strain-gaged cantilever beam was mounted on the exciter system base, and attached to the platform in use, to provide a signal proportional to displacement. Since displacement was fixed, this signal was used primarily as a phase reference.

III. TEST PROCEDURE

Before initiating the tests, the dynamometer system was calibrated by application of known weights. F_x, F_y, and M_z were calibrated by pulling horizontally on the tank platform in the X and Y directions (eccentrically for the M_z calibration). F_z was calibrated by stacking weights on top of the tank. M_x and M_y were calibrated by stacking weights on the tank platform, away from the center of the tank.

Table I outlines the test sequence. This sequence was set up to minimize the number of configuration changes, as changing from α to β tilt (see Figure 1) required moving the tank braces, changing from X to Y translation required moving the turntable, and changing from translation to rotation required changing the mode of excitation. Each of these changes resulted in the need for altering the balance arm masses for the best inertia cancellation.

Each test number corresponds to a given liquid height in the tank, for each configuration. Although not indicated in Table I, three runs were made for each test number, one at each of the first three natural liquid resonant modes. Three excitation directions were used; X translation, Y translation, and rotation (Ω) about the Z-axis. Three tank orientations were tested in each excitation direction; vertical, tilted 10° along the long, or X-axis (β), and tilted 10° along the short, or Y-axis (α). This results in nine configurations. Since five liquid heights were run at each configuration, a total of 45 tests (135 runs) was made at a single excitation amplitude.

Before each series of five tests at one configuration was made, a series of records was made with the empty tank at several frequencies covering the test frequency range, with each of the six force and moment channels recording the residual inertia signals associated with that configuration.

The tank was then filled to the initial depth, and the first natural liquid resonance established. A record of the forces and moments while at (or near) resonance during the forced excitation was made. The exciter platform was then stopped at the end of a stroke, and a record made of the decaying forces and moments on all channels.

A similar procedure was used at the second and third natural liquid modes, to complete the test. Liquid was then added to the tank to the next height, and the above procedure repeated until all five liquid heights had been run at each combination of excitation and tank tilt.

TABLE I

TEST SEQUENCE MATRIX

Test No.	Direction of Excitation	α (Deg.)	β (Deg.)	Direction of Braces	Liquid Height ($h/2r_o$)
1	X	0	0	β	0.25
2	X	0	0	β	0.50
3	X	0	0	β	0.75
4	X	0	0	β	1.00
5	X	0	0	β	1.50
6	X	0	10	β	0.25
7	X	0	10	β	0.50
8	X	0	10	β	0.75
9	X	0	10	β	1.00
10	X	0	10	β	1.50
11	Y	0	10	β	0.25
12	Y	0	10	β	0.50
13	Y	0	10	β	0.75
14	Y	0	10	β	1.00
15	Y	0	10	β	1.50
16	Y	0	0	β	0.25
17	Y	0	0	β	0.50
18	Y	0	0	β	0.75
19	Y	0	0	β	1.00
20	Y	0	0	β	1.50
21	Y	10	0	α	0.25
22	Y	10	0	α	0.50
23	Y	10	0	α	0.75
24	Y	10	0	α	1.00
25	Y	10	0	α	1.50

TABLE I

TEST SEQUENCE MATRIX (Cont'd.)

Test No.	Direction of Excitation	α (Deg.)	β (Deg.)	Direction of Braces	Liquid Height ($h/2\,r_o$)
26	X	10	0	α	0.25
27	X	10	0	α	0.50
28	X	10	0	α	0.75
29	X	10	0	α	1.00
30	X	10	0	α	1.50
31	Ω	0	0	α	0.25
32	Ω	0	0	α	0.50
33	Ω	0	0	α	0.75
34	Ω	0	0	α	1.00
35	Ω	0	0	α	1.50
36	Ω	10	0	α	0.25
37	Ω	10	0	α	0.50
38	Ω	10	0	α	0.75
39	Ω	10	0	α	1.00
40	Ω	10	0	α	1.50
41	Ω	0	10	β	0.25
42	Ω	0	10	β	0.50
43	Ω	0	10	β	0.75
44	Ω	0	10	β	1.00
45	Ω	0	10	β	1.50

The configuration was then changed by either tilting the tank or switching the excitation direction, and the next five tests made. This procedure was followed until all the tests at one amplitude were completed.

Data reduction was fairly straightforward. Where the decay records were adequate, the frequency was determined from them, rather than from the record of forced response. It was almost impossible to set the forcing frequency exactly on a liquid resonance, as this setting is dependent on the judgement of the operator and ability of the equipment to hold a set frequency. The low damping inherent in the unbaffled tank further complicated the setting, as it required a substantial time to settle into reasonably consistent liquid surface motion after adjusting the frequency. The set frequencies for forced excitation were therefore as much as 0.05 Hz off the actual resonance as determined from the decay records.

Logarithmic damping decrements were calculated from the decay records, while the forces and moments were determined from the records made during forced excitation. Phase relations relative to displacement were also obtained from these records.

Attempts were made during the initial tests to measure the liquid amplitudes at the tank wall, but were abandoned as the liquid surface motions at resonance were simply too inconsistent for reliable measurement. Not only were the amplitudes at the wall inconsistent, but there was always a tendency to "swirl"; that is, the peak waves would progressively rotate about the perimeter of the tank, sometimes changing the direction of this rotation.

All tests were made with tap water in the tank. An initial study had been made to find a test liquid which would have a nearly zero contact angle with the acrylic tank wall, without softening or crazing the acrylic. This study had been narrowed down to heptane, as the liquid which showed an essentially zero contact angle and the least measurable effect on samples of acrylic submerged in it for periods up to two weeks. Heptane is, however, a hazardous liquid to handle in the lab, and its use would have resulted in extra time consumed during the tests for special handling.

Accordingly, a short series of tests was made to compare damping ratios obtained with heptane and water. The result of these tests indicated the differences to be within the data scatter. This is not surprising, for unbaffled tanks of this size. Water was therefore used for the tests, in the interest of saving time and for safer handling.

IV. RESULTS

A. General

A compilation of the measured loads, frequencies, and damping factors is shown in Table II. All data is shown in both the SI system and in English units. Actual measurements were made in the English Unit system and converted to SI Units.

No plots of the load measurements were attempted, as the data are far too scattered. The loads are highly dependent on how nearly the forced excitation is to an actual resonance in a system as lightly damped as the one under study here. The only value to be derived from these measurements is as an indication of the order of magnitude of the peak loads at or near the first resonance. At the higher modes, in particular, the frequencies as set for maximum surface amplitude appear in several cases to have actually been near the point at which the phase of the loads shifts through 180°, resulting in very low measured loads.

It is interesting to note, from Table II, that in many of the cases where forces were measured along the vertical (Z) axis, this force (F_z) occurred at twice the excitation frequency. The phase angle associated with this force is the angle by which the first positive F_z peak lagged the positive displacement peak.

Plots of the measured frequencies versus liquid height are shown in Figures 4 through 6. Plots of the logarithmic decrements, where obtainable from the decay measurements, are shown in Figures 7 through 9. Here, again, few measurements were obtained in the second and third modes because of either noise on the traces or due to the fact that the loads were initially near zero. Decrements were obtainable from nearly all the first mode decay records, except for the lowest liquid level.

At the lowest level, the liquid surface was barely above the joint in the two hemispherical bottom sections. The liquid surface behavior was more erratic, and the load signals considerably more distorted, than at the higher levels, the distortion carrying over to the decay records in some cases.

Figures 10 through 12 are photographs of the liquid surface motions representative of all the combinations of excitation and tank tilt. All photographs were made at a liquid height to tank short diameter ratio of 0.75.

TABLE II--SUMMARY OF DATA

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2x_o$ = 8.51 mm (0.0335 in.) $\alpha = \beta = 0^\circ$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
1	1	0.68	0.274	0.133 (0.03) $\angle 0^\circ$								0.30 (2.64) $\angle 90^\circ$			
	2	1.76	1.84	2.18 (0.49) $\angle 0^\circ$											
	3														
2	1	0.967	0.555	3.43 (0.77) $\angle 95^\circ$	0.044							1.56 (13.85) $\angle 95^\circ$	0.0485		
	2	2.03	2.44	6.09 (1.37) $\angle 139^\circ$	0.0219							0.37 (3.3) $\angle 319^\circ$			
	3	2.905	5.02	1.73 (0.39) $\angle 0^\circ$											
3	1	1.153	0.652	17.3 (3.90) $\angle 89^\circ$	0.046							3.86 (34.2) $\angle 89^\circ$	0.043		
	2	2.13	2.70	16.3 (3.67) $\angle 94^\circ$	0.025							1.08 (9.57) $\angle 94^\circ$	0.028		
	3	3.19	6.06	3.79 (0.83) $\angle 180^\circ$											
4	1	1.23	0.899	27.2 (6.12) $\angle 104^\circ$	0.0383							6.53 (57.8) $\angle 104^\circ$	0.0391		
	2	2.13	2.70	3.47 (0.78) $\angle 143^\circ$	0.0228							0.81 (7.15) $\angle 143^\circ$			
	3	3.20	6.08	3.96 (0.89) $\angle 180^\circ$								1.04 (9.25) $\angle 180^\circ$			
5	1	1.27	0.956	13.5 (3.04) $\angle 146^\circ$	0.0464							7.29 (64.5) $\angle 146^\circ$	0.051		
	2	2.14	2.74	6.81 (1.53) $\angle 110^\circ$	0.0315							2.30 (20.4) $\angle 110^\circ$	0.0216		
	3	3.88	8.95	13.9 (3.13) $\angle 0^\circ$								3.85 (34.1) $\angle 0^\circ$			

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2x_o$ = 8.51 mm (0.0335 in.) $\alpha = 0^\circ$, $\beta = 10^\circ$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
6	1	0.645	0.247	0.75 (0.168) $\angle 0^\circ$											
	2	1.765	1.85	2.45 (0.55) $\angle 123^\circ$										0.03 (0.28) $\angle 0^\circ$	
	3	3.00	5.33	0.76 (0.17) $\angle 180^\circ$										0.20 (1.80) $\angle 0^\circ$	
7	1	0.975	0.565	3.74 (0.84) $\angle 14^\circ$	0.0367							1.20 (10.6) $\angle 14^\circ$	0.0417		
	2	2.00	2.38	0.46 (0.104) $\angle 180^\circ$	0.0286									0.03 (0.26) $\angle 0^\circ$	
	3	2.89	4.97	2.89 (0.65) $\angle 65^\circ$		1.60 (0.36) $\angle 180^\circ$		3.78 (0.85) $\angle 180^\circ$				0.88 (7.75) $\angle 30^\circ$		0.59 (5.2) $\angle 180^\circ$	
8	1	1.13	0.71	14.6 (3.28) $\angle 129^\circ$	0.0182							3.12 (27.6) $\angle 129^\circ$	0.018		
	2	2.08	2.59	0.36 (0.08) $\angle 0^\circ$	0.0176							0.69 (6.07) $\angle 0^\circ$			
	3	2.88	4.88	3.02 (0.68) $\angle 0^\circ$											
9	1	1.20	0..856	23.9 (5.37) $\angle 40^\circ$	0.0252							4.69 (41.5) $\angle 40^\circ$	0.0282	0.52 (4.6) $\angle 199^\circ$	0.0336
	2	2.08	2.58	4.49 (1.01) $\angle 125^\circ$	0.0137			28.6* (6.43) $\angle 90^\circ$				0.13 (1.16) $\angle 150^\circ$		0.24 (2.12) $\angle 344^\circ$	
	3	2.91	5.05	2.67 (0.60) $\angle 0^\circ$				0.37 (0.083) $\angle 180^\circ$						0.85 (7.5) $\angle 180^\circ$	
10	1	1.254	0.935	2.72 (6.11) $\angle 53^\circ$	0.036	1.03 (0.231) $\angle 271^\circ$		Not Meas.*		0.48 (4.28) $\angle 38^\circ$		9.15 (81.0) $\angle 49^\circ$	0.0345	0.85 (7.50) $\angle 245^\circ$	0.0295
	2	2.105	2.62	5.52 (1.24) $\angle 98^\circ$	0.015			Not Meas.*				2.48 (21.9) $\angle 113^\circ$	0.0246	0.47 (4.2) $\angle 306^\circ$	
	3	2.90	5.00	9.15 (2.06) $\angle 0^\circ$								1.72 (15.25) $\angle 21^\circ$		0.19 (1.7) $\angle 0^\circ$	

*F_z occurs at twice the excitation frequency.

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2y_o$ = 8.51 mm (0.0335 in.) $\alpha = 0$, $\beta = 10^\circ$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
11	1	1.397	1.16			6.85 (1.54) $\angle 164^\circ$	0.0345			0.84 (7.4) $\angle 188^\circ$	0.0213			0.46 (4.06) $\angle 182^\circ$	0.0342
	2														
	3	3.14	5.85			5.65 (1.27) $\angle 180^\circ$									
12	1	1.55	1.43			17.0 (3.82) $\angle 7^\circ$	0.0099	2.32* (0.52) $\angle 0^\circ$		1.99 (17.65) $\angle 11^\circ$	0.0147			0.09 (8.0) $\angle 8^\circ$	0.0212
	2	2.22	2.92			3.18 (0.715) $\angle 135^\circ$									
	3	2.98	5.27			4.00 (0.90) $\angle 0^\circ$									
13	1	1.65	1.61			3.45 (0.775) $\angle 163^\circ$	0.0104	9.35* (2.10) $\angle 161^\circ$		4.75 (42.1) $\angle 161^\circ$	0.0117			1.60 (14.15) $\angle 163^\circ$	0.0121
	2	2.20	2.86			1.96 (0.44) $\angle 102^\circ$		2.72* (0.61) $\angle 217^\circ$						0.13 (1.13) $\angle 125^\circ$	
	3														
14	1	1.67	1.66			33.8 (7.60) $\angle 146^\circ$	0.0118	20.0* (4.50) $\angle 147^\circ$		9.85 (87.2) $\angle 147^\circ$	0.0136			1.85 (16.4) $\angle 151^\circ$	0.007
	2	2.20	2.90			1.65 (0.37) $\angle 180^\circ$								0.14 (1.22) $\angle 180^\circ$	
	3	2.97	5.24			1.02 (0.23) $\angle 0^\circ$									
15	1	1.67	1.66			39.1 (8.8) $\angle 168^\circ$	0.0118	13.5* (3.04) $\angle 167^\circ$		13.4 (119.0) $\angle 167^\circ$	0.0114			1.98 (17.5) $\angle 168^\circ$	0.0109
	2	2.67	4.23			11.3 (2.55) $\angle 0^\circ$				0.13 (1.11) 0°				0.59 (5.25) $\angle 0^\circ$	
	3														

*F_z occurs at twice the excitation frequency.

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2y_o$ = 8.51 mm (0.0335 in.) $\alpha = \beta = 0$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
16	1	1.38	1.13			9.65 (2.17) $\angle 80^\circ$	0.0334	14.9* (3.34) $\angle 90^\circ$		1.01 (8.9) $\angle 106^\circ$				0.84 (7.4) $\angle 70^\circ$	
	2														
	3														
17	1	1.53	1.385			11.3 (2.53) $\angle 180^\circ$	0.0157			1.62 (14.3) $\angle 180^\circ$	0.0201				
	2	2.14	2.72			1.20 (0.27) $\angle 0^\circ$									
	3	2.97	5.21			1.78 (0.40) $\angle 0^\circ$									
18	1	1.64	1.60			27.6 (6.20) $\angle 173^\circ$	0.01105	5.78* (1.30) $\angle 0^\circ$		3.80 (33.6) $\angle 178^\circ$	0.00885			3.20 (28.3) $\angle 179^\circ$	0.0170
	2	2.20	2.88			1.20 (0.27) $\angle 0^\circ$								0.15 (1.29) $\angle 0^\circ$	
	3	2.90	5.00			2.84 (0.64) $\angle 0^\circ$								0.25 (2.20) $\angle 0^\circ$	
19	1	1.667	1.64			30.0 (6.75) $\angle 209^\circ$	0.0108	8.65* (1.94) $\angle 222^\circ$		6.04 (53.5) $\angle 222^\circ$	0.0100	0.80 (7.1) $\angle 330^\circ$		4.56 (40.4) $\angle 204^\circ$	0.0125
	2	2.19	2.85												
	3	2.92	5.07			7.75 (1.74) $\angle 0^\circ$								0.67 (5.9) $\angle 0^\circ$	
20	1	1.67	1.66			36.5 (8.20) $\angle 174^\circ$	0.01067	11.7* (2.64) $\angle 0^\circ$		12.1 (107.2) $\angle 175^\circ$	0.01192			5.25 (46.5) $\angle 176^\circ$	0.0065
	2	2.44	3.52			6.00 (1.35) $\angle 0^\circ$								0.68 (6.0) $\angle 0^\circ$	
	3	2.91	5.00			10.3 (2.32) $\angle 0^\circ$								0.97 (8.6) $\angle 0^\circ$	

*F_z occurs at twice the excitation frequency.

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2y_o$ = 8.51 mm (0.0335 in.) $\alpha = 10^\circ$, $\beta = 0$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
21	1	1.43	1.21												
	2														
	3														
22	1	1.54	1.41			26.7 (6.0) ∠170°	0.0075	5.45* (1.225) ∠168°		2.92 (25.8) ∠168°	0.0102			1.21 (10.7) ∠170°	0.0086
	2	2.12	2.65			3.47 (0.78) ∠0°								0.04 (0.34) ∠180°	
	3	2.90	4.98			0.18 (0.04) ∠0°								0.21 (1.9) ∠0°	
23	1	1.63	1.58			45.8 (10.3) ∠148°	0.01173	15.9* (3.58) ∠151°		5.95 (52.6) ∠151°	0.0116			2.28 (20.2) ∠150°	0.0136
	2	2.26	3.03			0.89 (0.02) ∠180°									
	3	3.02	5.40			3.20 (0.72) ∠180°								0.32 (2.8) ∠0°	
24	1	1.648	1.61			53.0 (11.9) ∠149°	0.0093	24.9* (5.60) ∠150°		10.2 (90.5) ∠150°	0.0113			2.50 (22.1) ∠150°	
	2	2.21	2.89			0.42 (0.095) ∠0°									
	3	2.99	5.32			2.04 (0.46) ∠0°								0.42 (3.75) ∠0°	
25	1	1.644	1.608			51.3 (11.53) ∠145°	0.01035	18.5 (4.16) ∠151°		17.2 (152.5) ∠151°	0.0079			2.56 (22.6) ∠148°	0.0096
	2	2.20	2.88			1.33 (0.30) ∠0°								0.31 (2.79) ∠0°	
	3	2.83	4.76			8.99 (2.02) ∠0°								0.21 (1.89) ∠0°	

*F_z occurs at twice the excitation frequency.

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2x_o$ = 8.51 mm (0.0335 in.) $\alpha = 10^\circ$, $\beta = 0$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
26	1														
	2	1.752	1.82	0.67 (0.015) $\angle 0^\circ$										0.26 (2.29) $\angle 0^\circ$	
	3	2.28	3.09	0.45 (0.10) $\angle 180^\circ$										0.36 (3.15) $\angle 0^\circ$	
27	1	0.956	0.542	4.25 (0.955) $\angle 126^\circ$	0.0533							1.72 (15.25) $\angle 127^\circ$	0.0314		
	2	2.00	2.38	2.04 (0.46) $\angle 180^\circ$	0.0213							0.55 (4.84) $\angle 180^\circ$		0.32 (2.84) $\angle 0^\circ$	
	3	2.58	3.96	1.20 (0.27) $\angle 0^\circ$										0.05 (0.40) $\angle 180^\circ$	
28	1	1.175	0.82	19.4 (4.35) $\angle 78^\circ$	0.0334							4.45 (39.4) $\angle 81^\circ$	0.0280	0.98 (8.7) $\angle 253^\circ$	
	2	2.13	2.70	5.20 (1.17) $\angle 0^\circ$	0.0312							0.66 (5.8) $\angle 0^\circ$		0.04 (0.34) $\angle 0^\circ$	
	3	2.50	3.70	9.20 (2.07) $\angle 0^\circ$										1.71 (15.1) $\angle 180^\circ$	
29	1	1.215	0.877	19.2 (4.32) $\angle 144^\circ$	0.0265							4.64 (41.0) $\angle 146^\circ$	0.0227	1.16 (10.25) $\angle 330^\circ$	0.0210
	2	2.14	2.73	0.40 (0.09) $\angle 180^\circ$										0.29 (2.60) $\angle 0^\circ$	
	3	2.89	4.94	7.55 (1.70) $\angle 0^\circ$										0.40 (3.5) $\angle 180^\circ$	
30	1	1.25	0.926	17.2 (3.87) $\angle 142^\circ$	0.0247							9.05 (80.1) $\angle 139^\circ$	0.0260	2.17 (19.20) $\angle 323^\circ$	0.0279
	2	2.10	2.61	4.62 (1.14) $\angle 87^\circ$								2.43 (21.5) $\angle 105^\circ$		0.53 (4.7) $\angle 282^\circ$	
	3	2.98	5.27	10.0 (2.25) $\angle 0^\circ$								1.82 (16.1) $\angle 0^\circ$		0.59 (5.2) $\angle 180^\circ$	

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2\,\Omega_o = 0.0225$ rad. $\alpha = \beta = 0$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	MOMENTS M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
31	1	1.612	1.55			91.5 (20.6) $\angle 180°$		3.52 (0.79) $\angle 0°$						0.56 (5.0) $\angle 0°$	
	2														
	3														
32	1	1.724	1.78	4.80 (1.08) $\angle 0°$				18.9 (4.25) $\angle 0°$						1.70 (15.0) $\angle 180°$	0.0288
	2	2.31	3.18	47.5 (10.7) $\angle 0°$				4.00 (0.90) $\angle 180°$						23.4 (207) $\angle 0°$	
	3														
33	1	1.83	1.99											11.0 (97.0) $\angle 0°$	0.00623
	2	2.45	3.54	6.80 (1.53) $\angle 0°$		3.56 (0.8) $\angle 180°$		25.6 (5.76) $\angle 180°$						56.7 (502) $\angle 180°$	
	3														
34	1	1.82	1.97	13.3 (0.30) $\angle 0°$		30.2 (6.8) $\angle 180°$		3.87 (0.87) $\angle 0°$		7.96 (70.5) $\angle 180°$				4.52 (40) $\angle 0°$	0.01134
	2	2.39	3.40			77.5 (17.4) $\angle 180°$		11.6 (2.6) $\angle 180°$				4.75 (42.1) $\angle 180°$		15.3 (135) $\angle 0°$	
	3														
35	1	1.83	1.99	6.75 (1.52) $\angle 180°$		62.7 (14.1) $\angle 180°$		2.27 (0.51) $\angle 0°$						1.58 (14.0 $\angle 0°$	0.01613
	2	2.42	3.48			127.0 (28.5) $\angle 180°$		0.45 (0.10) $\angle 180°$		10.8 (95.8) $\angle 180°$				5.65 (50.0) $\angle 0°$	
	3														

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2\,\Omega_o = 0.0225$ rad. $\alpha = 10°$, $\beta = 0°$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
36	1	1.77	1.85			28.2 (6.32) $\angle 0°$		25.7 (5.76) $\angle 188°$						44.0 (390) $\angle 194°$	
	2	2.29	3.12					13.5 (3.03) $\angle 180°$						3.5 (31) $\angle 0°$	
	3														
37	1	1.695	1.705	0.27 (0.06) $\angle 0°$		5.51 (1.24) $\angle 0°$		3.42 (0.77) $\angle 0°$		0.14 (1.2) $\angle 0°$				8.81 (78) $\angle 180°$	0.0438
	2														
	3	2.94	5.15					4.45 (1.0) $\angle 0°$						19.7 (174) $\angle 180°$	
38	1	1.82	1.97	5.65 (1.27) $\angle 0°$		35.1 (7.90) $\angle 180°$						0.05 (0.40) $\angle 180°$		4.52 (40) $\angle 0°$	0.0185
	2	1.786	1.90	16.6 (3.73) $\angle 0°$										27.2 (240) $\angle 0°$	0.0218
	3	2.83	4.75	27.1 (6.10) $\angle 0°$				4.00 (0.90) $\angle 180°$						16.0 (142) $\angle 180°$	
39	1	1.802	1.93	2.02 (0.46) $\angle 0°$		29.7 (6.67) $\angle 180°$		9.42* (2.12) $\angle 0°$						25.3 (224) $\angle 180°$	0.0084
	2	2.37	3.33	0.133 (0.03) $\angle 0°$		79.7 (17.90) $\angle 180°$		22.8 (5.11) $\angle 180°$						42.0 (372) $\angle 180°$	
	3	2.89	4.97	17.6 (3.95) $\angle 180°$		123.0 (27.60) $\angle 180°$		19.4 (4.35) $\angle 180°$						56.1 (497) $\angle 180°$	
40	1	1.805	1.93	12.7 (2.85) $\angle 180°$		9.52 (2.14) $\angle 0°$								18.8 (166) $\angle 180°$	0.01605
	2	2.38	3.36	0.53 (0.12) $\angle 0°$		4.89 (1.10) $\angle 180°$				1.38 (12.2) $\angle 180°$		6.27 (55.5) $\angle 180°$		65.3 (578) $\angle 180°$	
	3	2.91	5.02	22.4 (5.03) $\angle 0°$				14.2 (3.20 $\angle 180°$						0.96 (8.51) $\angle 180°$	

*F_z occurs at twice the excitation frequency.

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2\,\Omega_o = 0.0225$ Rad. $\alpha = 0$, $\beta = 10°$

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
41	1	1.715	1.758												0.0267
	2														
	3														
42	1	1.73	1.78												0.01384
	2	2.28	3.09												
	3	2.88	4.94												
43	1	1.82	1.96			FORCE DATA NOT REDUCIBLE									0.01585
	2	2.64	4.15												
	3	2.86	4.85												
44	1	1.833	2.00												Beating with long. mode
	2	2.42	3.48												
	3	2.82	4.71												
45	1	1.765	1.85												Beating with long. mode
	2	2.19	2.85												Beating with long. mode
	3	2.75	4.48												

TABLE II--SUMMARY OF DATA (Cont'd.)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2x_o$ or $2y_o$ = 17.5 mm (0.069 in.)

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	MOMENTS M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
88 (Same as Test 3 except $2x_o$ = 17.5 mm, 0.069 in.)	1	1.158	0.795	23.4 (5.25) $\angle 67^\circ$	0.0300	3.65 (0.82 $\angle 240^\circ$						6.58 (58.3) $\angle 61^\circ$	0.0396	0.58 (5.10) $\angle 261^\circ$	
	2	2.13	2.70	10.6 (2.39) $\angle 164^\circ$		0.31 (0.07) $\angle 180^\circ$						0.88 (7.80) $\angle 142^\circ$		0.11 (0.95) $\angle 0^\circ$	
	3	2.93	5.10	1.91 (0.43) $\angle 180^\circ$		0.045 (0.01) $\angle 0^\circ$						1.85 (16.40) $\angle 0^\circ$		0.08 (0.70) $\angle 180^\circ$	
73 (Same as Test 18 except $2y_o$ = 17.5 mm, 0.069 in.)	1	1.667	1.643	1.02 (0.23) $\angle 180^\circ$		20.5 (4.61) $\angle 180^\circ$	0.01186			3.26 (28.9) $\angle 180^\circ$	0.00978				
	2	2.20	2.88	0.40 (0.09) $\angle 180^\circ$		5.03 (1.13) $\angle 180^\circ$									
	3	2.93	5.09												
63 (Same as Test 28 except $2x_o$ = 17.5 mm, 0.069 in.)	1	1.146	0.78	14.0 (3.14) $\angle 0^\circ$	0.0361	2.00 (0.45) $\angle 180^\circ$						3.32 (29.4) $\angle 0^\circ$	0.0340	0.44 (3.88) $\angle 180^\circ$	
	2	2.10	2.61	2.05 (0.46) $\angle 180^\circ$	0.0326							1.50 (13.25) $\angle 180^\circ$	0.0276		
	3	2.93	5.10	0.62 (0.14) $\angle 180^\circ$		1.92 (0.43) $\angle 180^\circ$		5.33 (1.20) $\angle 0^\circ$				0.76 (6.75) $\angle 0^\circ$			
68 (Same as Test 23 except $2y_o$ = 17.5 mm, 0.069 in.)	1	1.655	1.62			28.0 (6.30) $\angle 180^\circ$	0.0112	7.25* (1.63) $\angle 0^\circ$		3.98 (35.3) $\angle 180^\circ$	0.01345				
	2	2.19	2.86	0.27 (0.06) $\angle 180^\circ$		7.02 (1.58) $\angle 180^\circ$									
	3	2.92	5.06												
78 (Same as Test 12 except $2y_o$ = 17.5 mm, 0.069 in.)	1	1.665				12.5 (2.81) $\angle 180^\circ$	0.01055	5.52* (1.24) $\angle 0^\circ$		2.87 (25.4) $\angle 180^\circ$	0.0126			2.10 (18.6) $\angle 180^\circ$	0.0155
	2	2.20		6.27 (1.41) $\angle 180^\circ$		10.9 (2.45) $\angle 180^\circ$									
	3	2.82		8.45 (1.90) $\angle 180^\circ$		4.49 (1.01) $\angle 180^\circ$									

*F_z occurs at twice the excitation frequency.

TABLE II--SUMMARY OF DATA (Concluded)

FORCE UNITS IN NEWTONS AND (POUNDS) - MOMENT UNITS IN NEWTON-METERS AND (INCH-POUNDS)

$2x_o$ = 17.5 mm (0.069 in.)

TEST NO.	RUN NO.	f_N (Hz)	$\frac{\omega_N^2 r_o}{g}$	FORCES						MOMENTS					
				F_x	λ_{F_x}	F_y	λ_{F_y}	F_z	λ_{F_z}	M_x	λ_{M_x}	M_y	λ_{M_y}	M_z	λ_{M_z}
83 (Same as Test 8 except $2x_o$ = 17.5 mm, 0.069 in.)	1	1.15		4.72 (1.06) $\angle 0^\circ$	0.0149	1.38 (0.31) $\angle 180^\circ$						1.62 (14.3) $\angle 0^\circ$	0.0137		
	2	2.09		9.25 (2.08) $\angle 180^\circ$								1.46 (12.9) $\angle 180^\circ$			
	3	2.89													

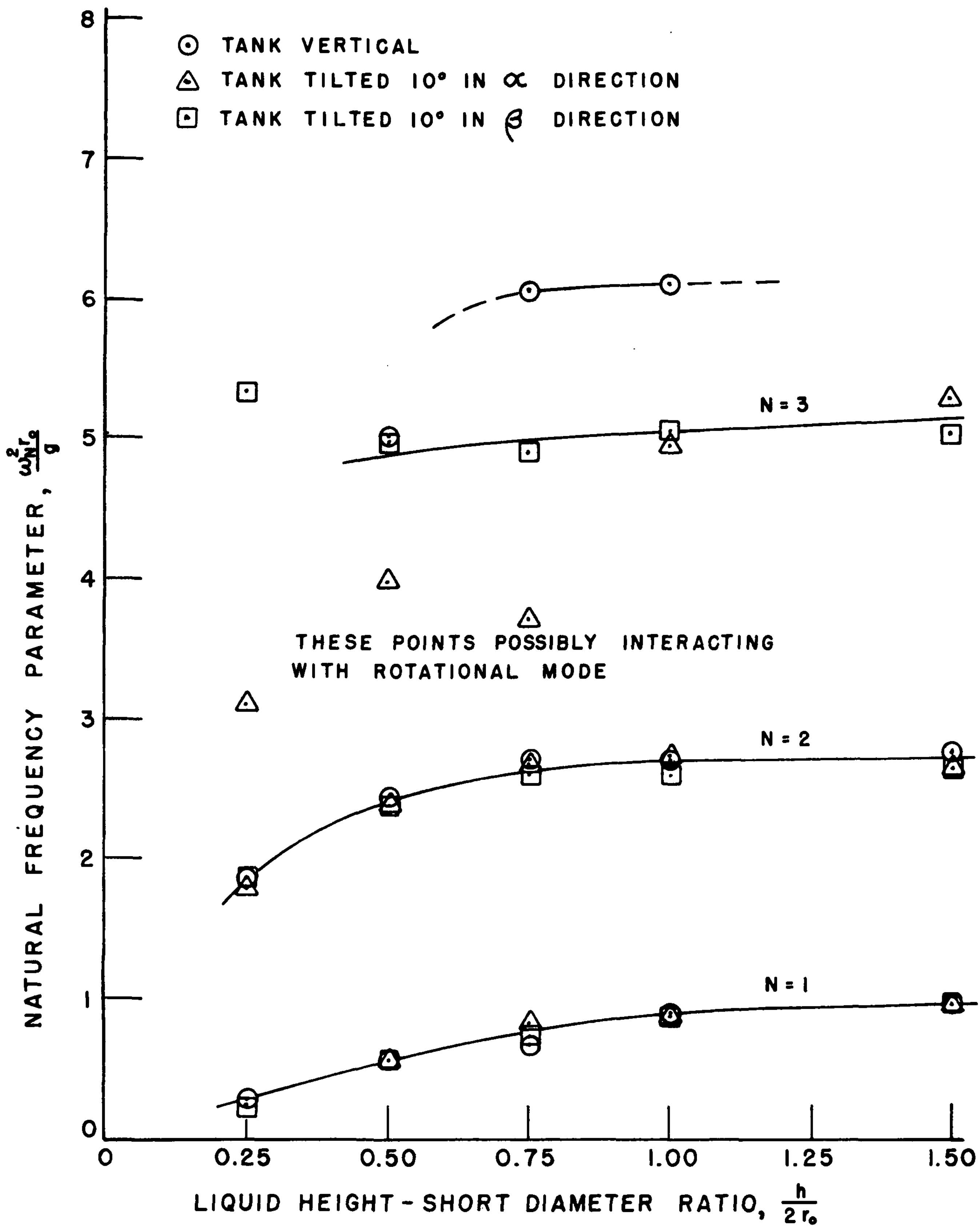


FIGURE 4. NATURAL FREQUENCY VARIATION WITH FILL HEIGHT-EXCITATION ALONG X-AXIS

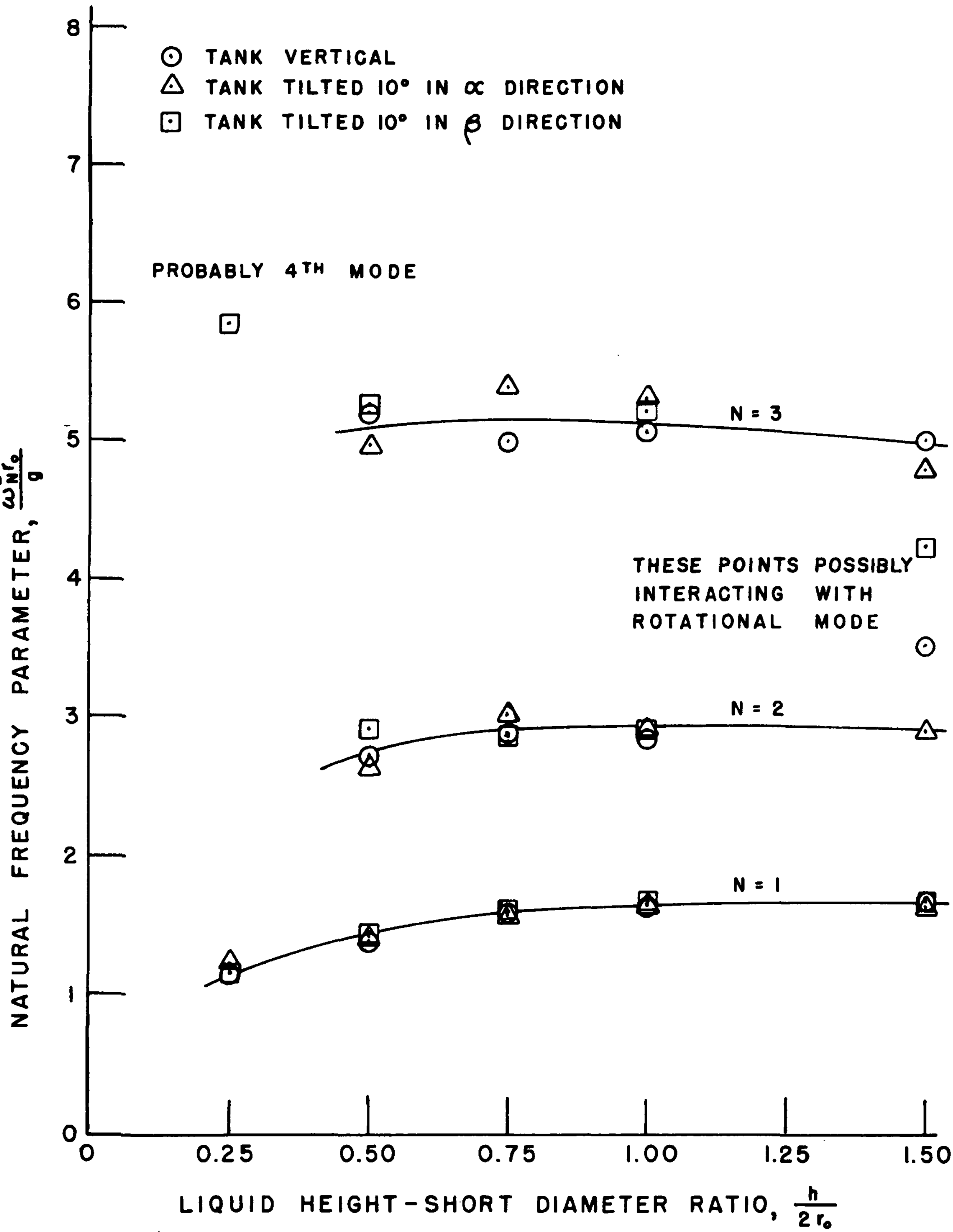


FIGURE 5. NATURAL FREQUENCY VARIATION WITH FILL HEIGHT-EXCITATION ALONG Y-AXIS

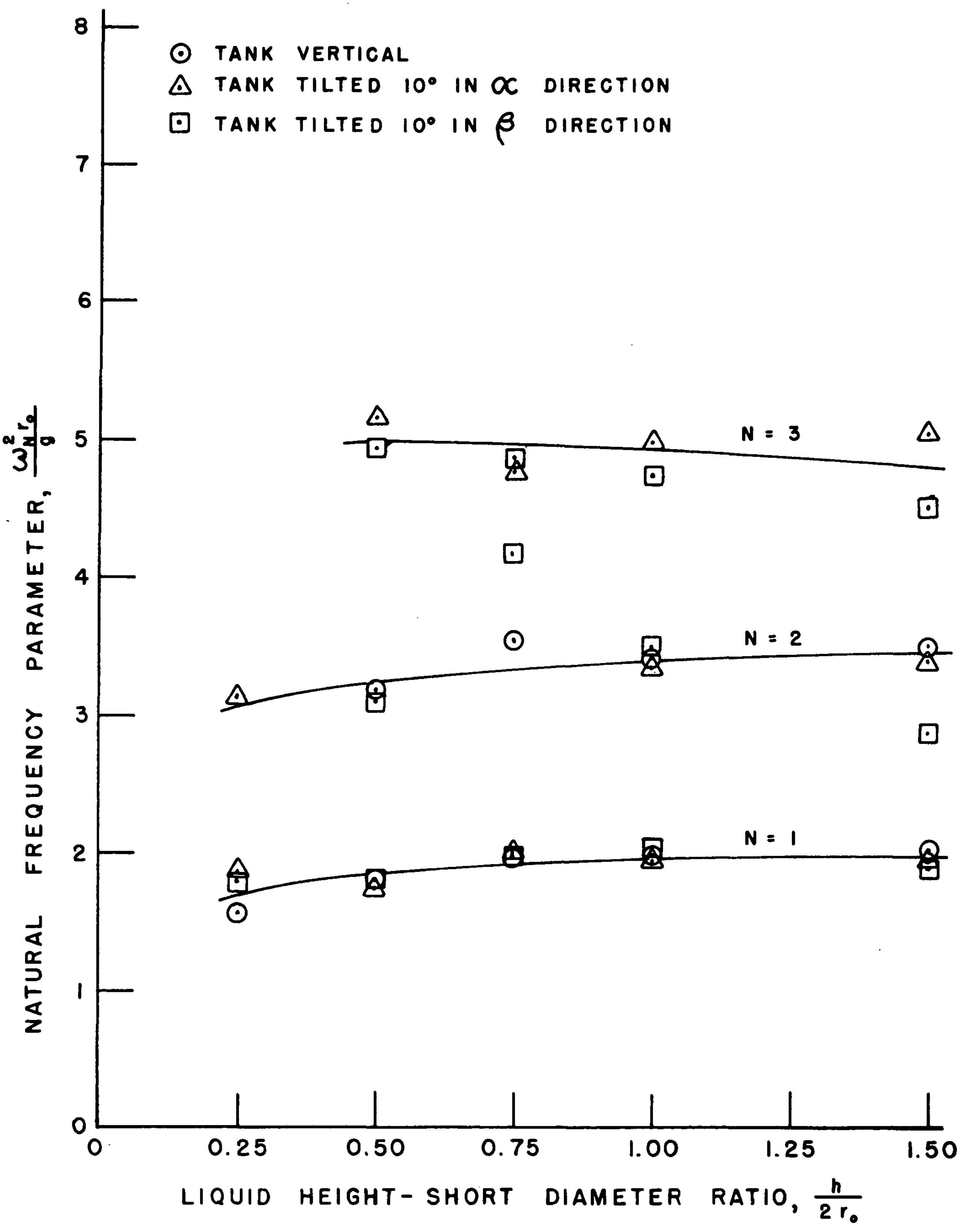


FIGURE 6. NATURAL FREQUENCY VARIATION WITH FILL HEIGHT-EXCITATION ABOUT Z-AXIS

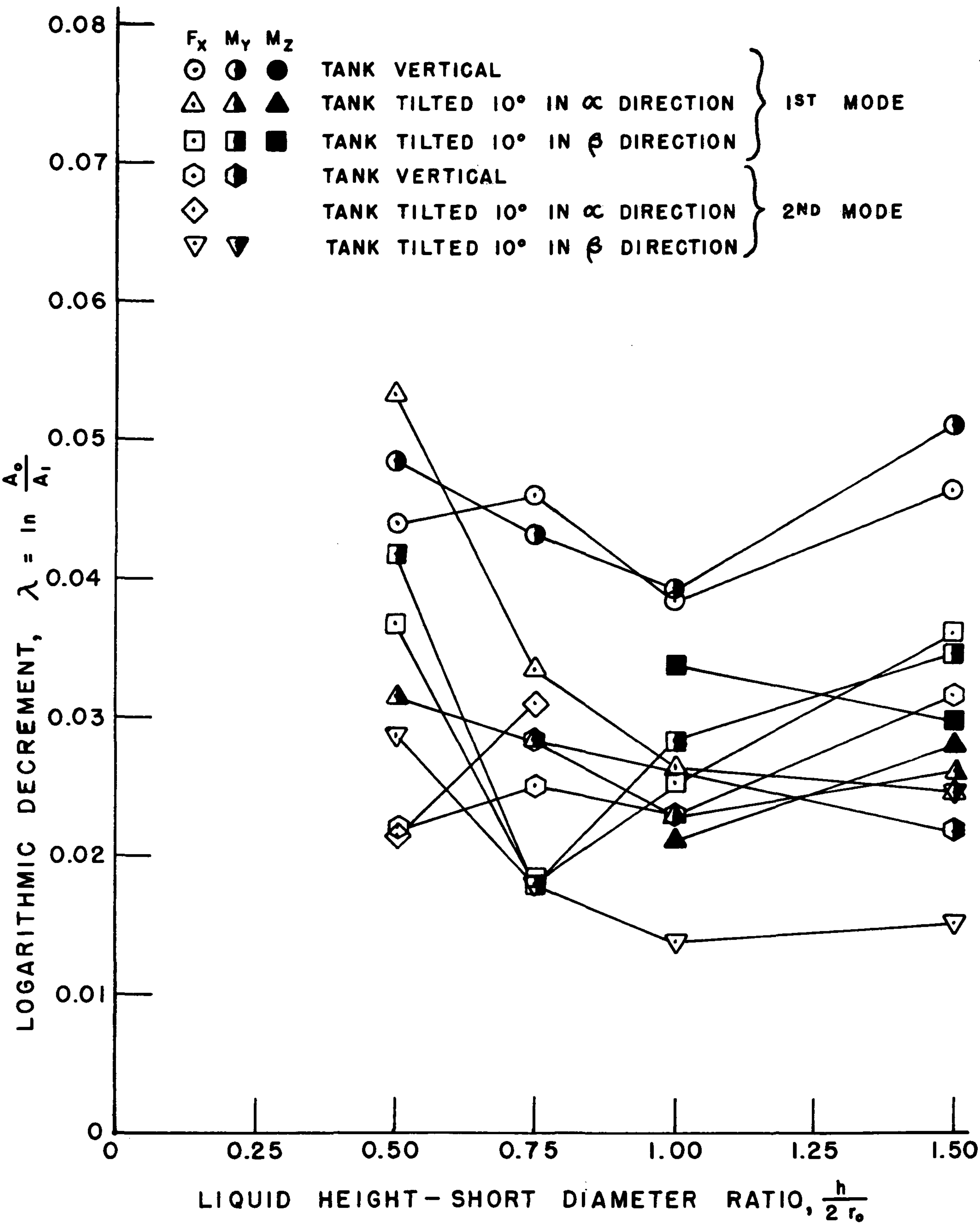


FIGURE 7. DAMPING VARIATION WITH FILL HEIGHT-EXCITATION ALONG X-AXIS

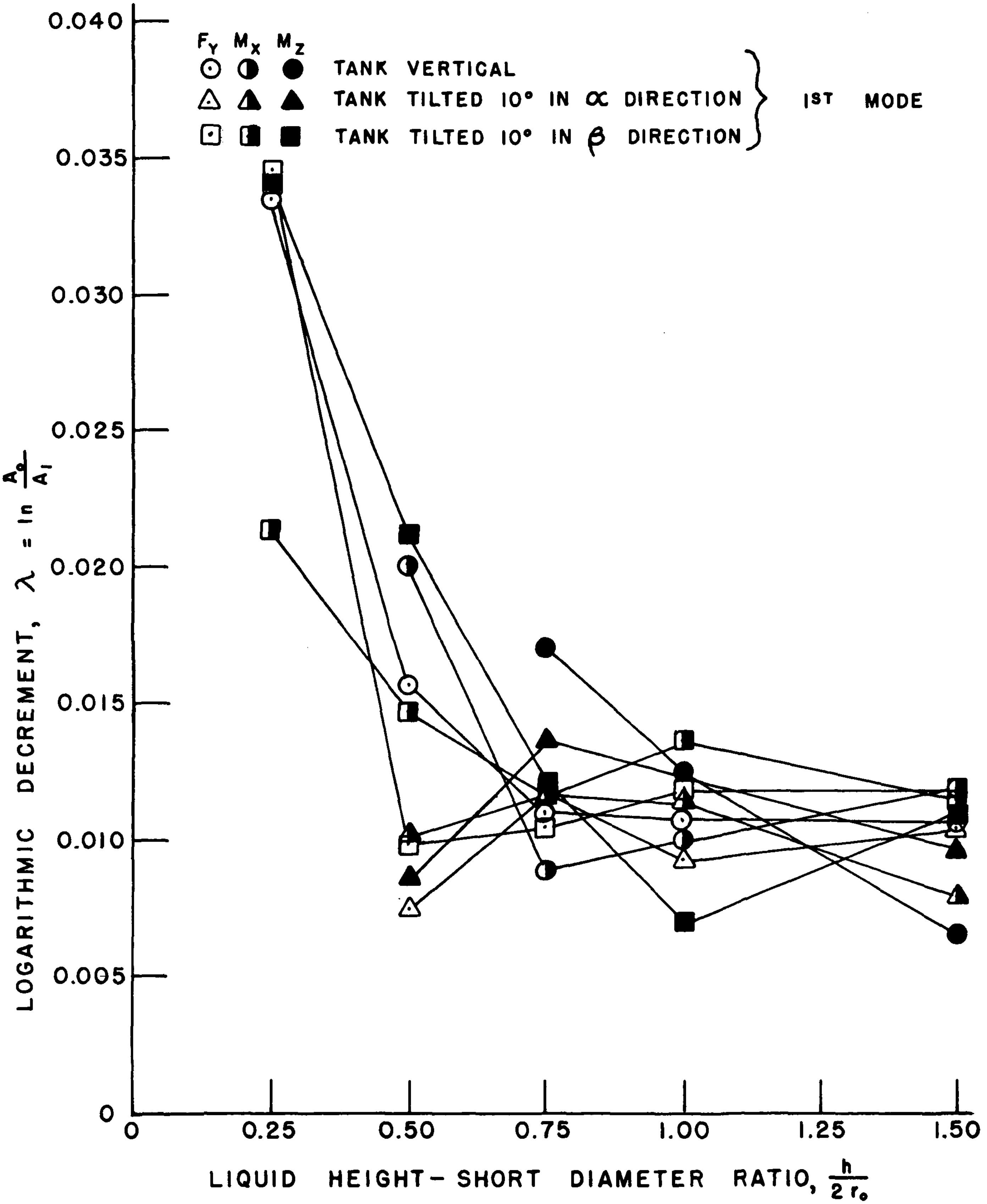


FIGURE 8. DAMPING VARIATION WITH FILL HEIGHT-EXCITATION ALONG Y-AXIS

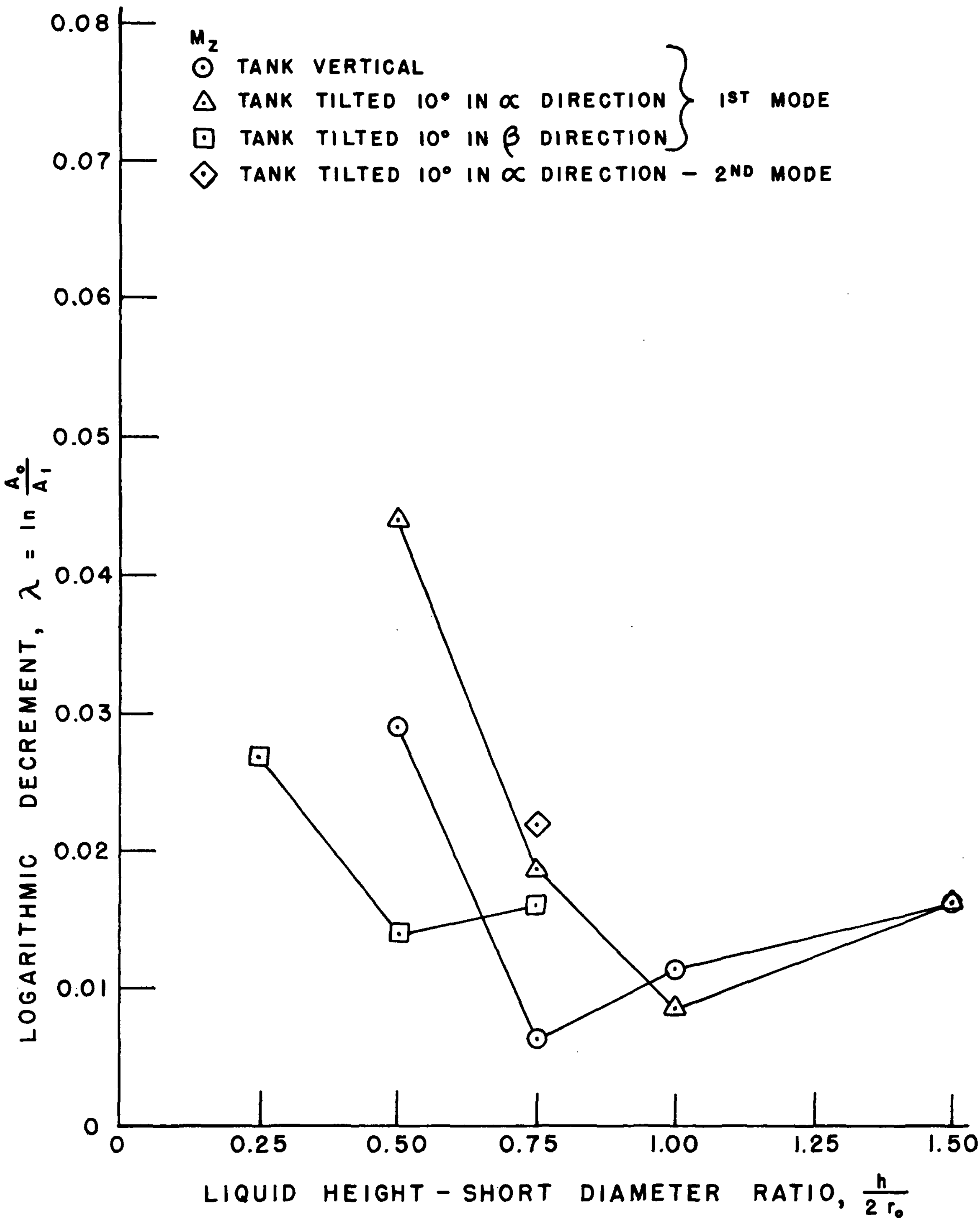
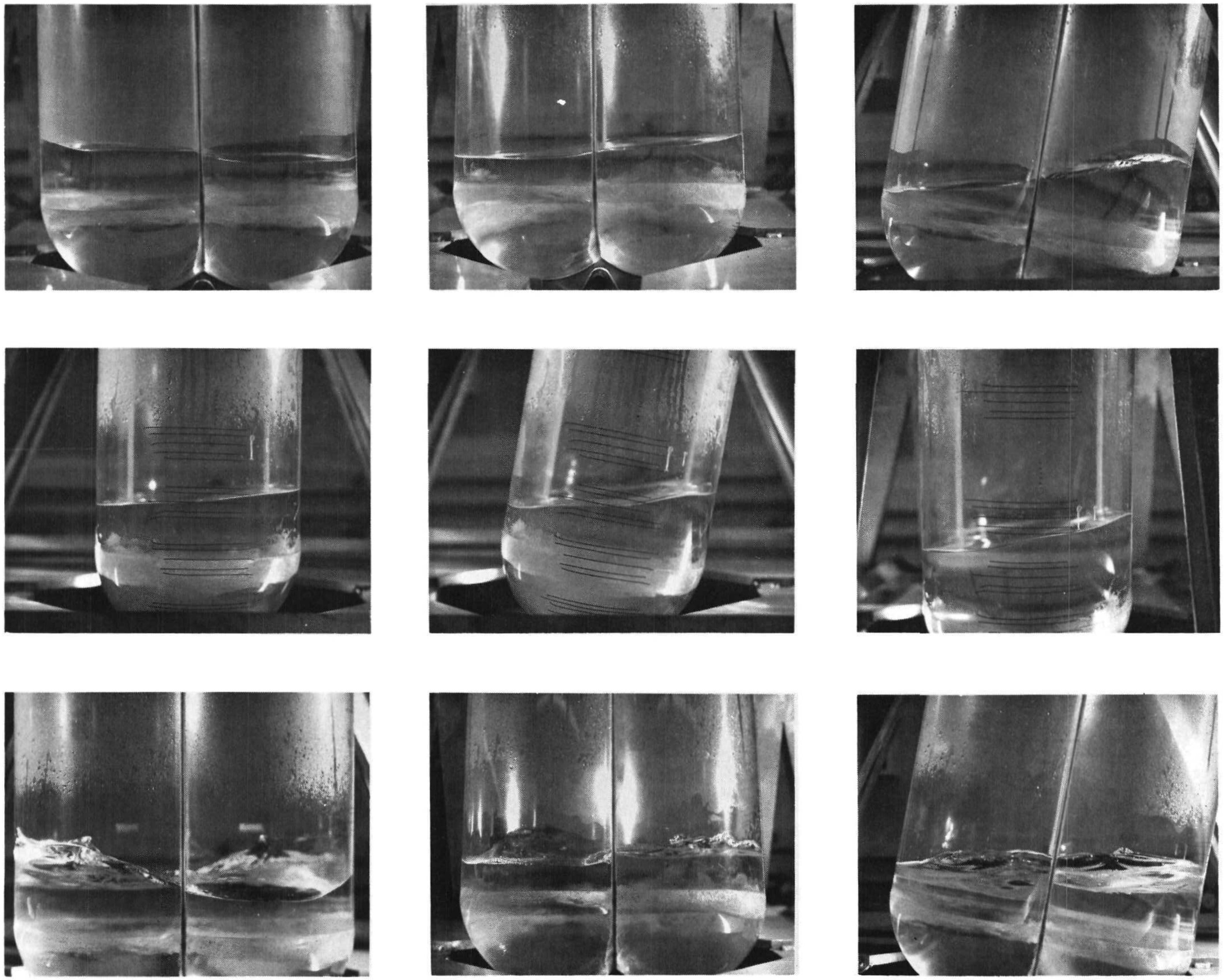


FIGURE 9. DAMPING VARIATION WITH FILL HEIGHT-EXCITATION ABOUT Z-AXIS

X Excit.

Y Excit.

Ω Excit.



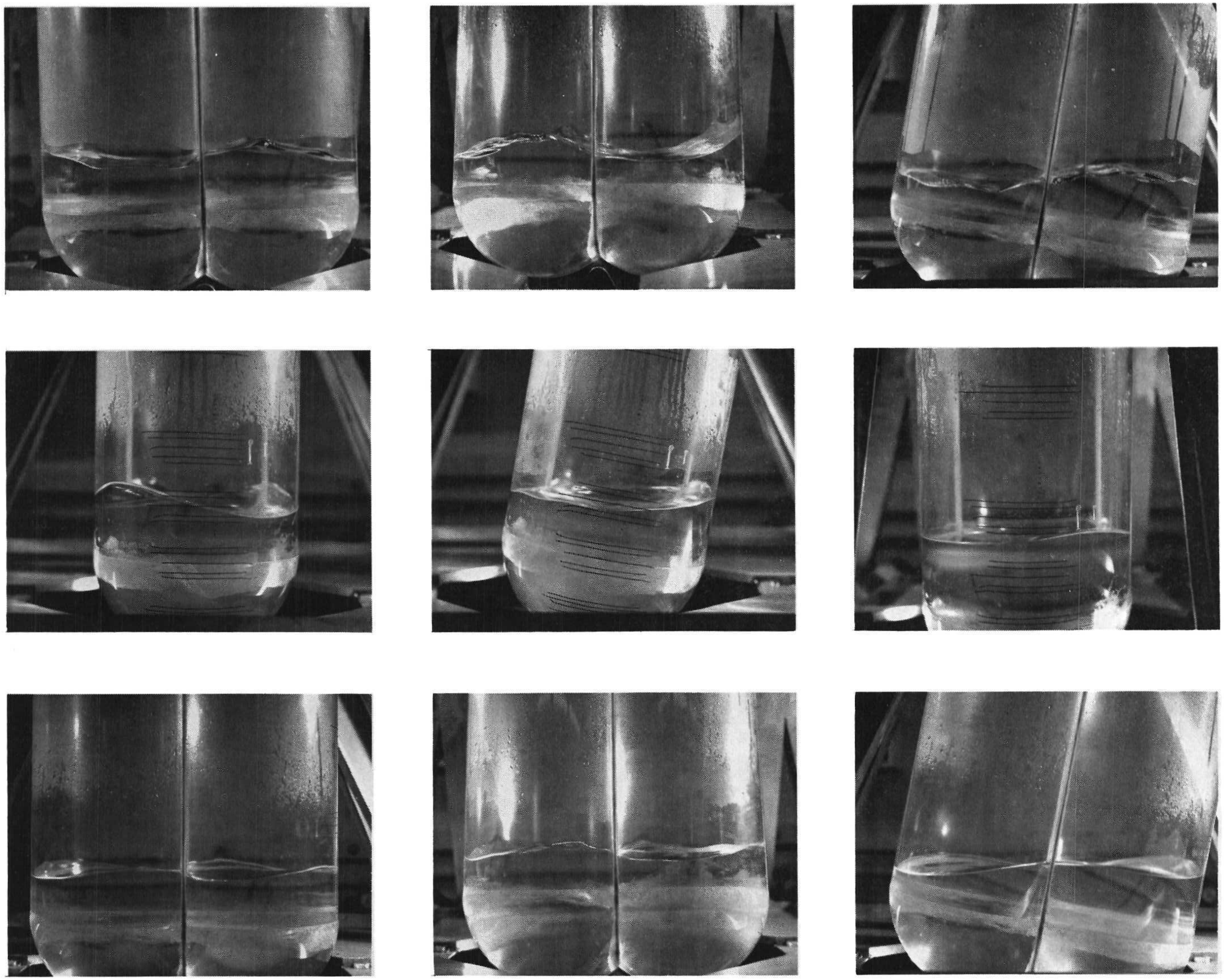
Vertical Tank

$\alpha = 10^\circ, \ \beta = 0$

$\alpha = 0 \quad \beta = 0^\circ$

FIGURE 10. FIRST MODE LIQUID SURFACE RESPONSE

X Excit. Y Excit. Ω Excit.



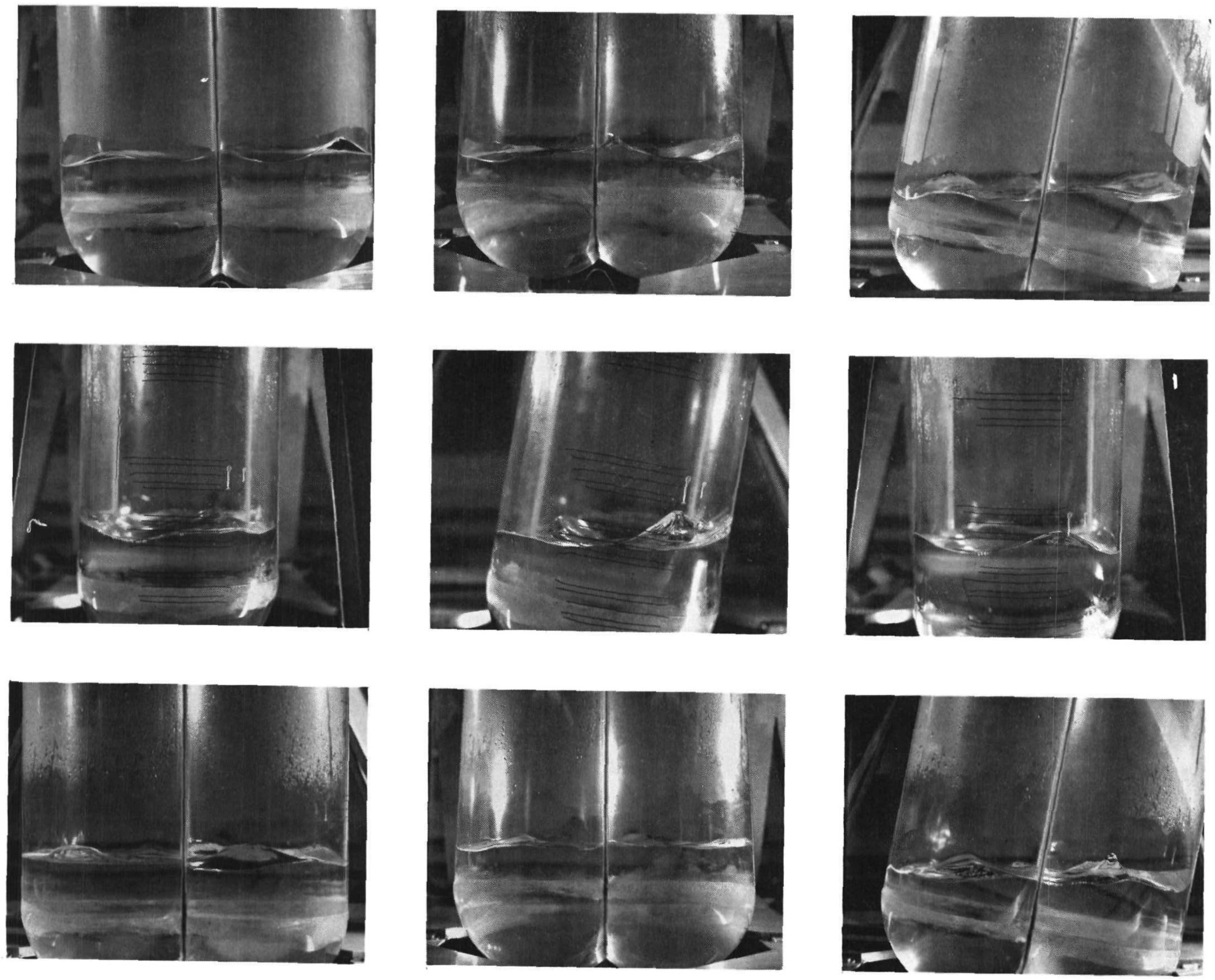
Vertical Tank | $\alpha = 10^\circ$, $\beta = 0$ | $\alpha = 0$, $\beta = 10^\circ$

FIGURE 11. SECOND MODE LIQUID SURFACE RESPONSE

X
Excit.

Y
Excit.

Ω
Excit.



Vertical Tank

$\alpha = 10^\circ, \quad \beta = 0$

$\alpha = 0, \quad \beta = 10^\circ$

FIGURE 12. THIRD MODE LIQUID SURFACE RESPONSE

Blank spaces in Table II indicate no data obtained, due either to inability to establish the mode, loads being too small to measure, or to noise or distortion of the trace.

Very little useable data were obtained from the rotational tests, especially with the tank tilted. Interactions with the translational modes resulted in considerable distortion of the data—indeed, it was very difficult to even establish frequencies which did not coincide with translational frequencies. In several instances, even where a resonance was established at an apparent rotational mode, the decay degenerated to one of the translational modes when the motion was stopped. With the tank tilted in the α-direction (along the short axis) the motion would fall into a mode corresponding to translation along the long (x) axis, while with the tank tilted in the β-direction (along the long axis) the motion would fall into a mode corresponding to translation along the short (y) axis.

A few of the tests were repeated at a different excitation amplitude (approximately half the initial rotational amplitude and twice the initial translational amplitudes). These tests yielded data which was little or no different from that at the initial amplitudes, within the data scatter, as may be seen by comparing the data on the last two pages of Table II with that for the corresponding tests at the lower amplitude. Therefore, the remaining tests of the originally scheduled final 45 were not made.

B. Frequencies and Liquid Surface Motion

Figures 4 and 5 indicate the first mode translational frequencies along the short (y) axis to be significantly higher (by about 32% at the higher levels than those along the long (x) axis), but by no means higher by the ratio of long to short "diameters" (67.5%). The second mode frequencies are only about 5% higher along the short axis than along the long axis, and the third mode frequencies are almost identical along both axes, for the tank tilted in either direction. Apparently the "pinched" portion at the juncture of the two halves of the tank restricts the transfer of liquid between halves so that they behave more or less as independent tanks of the short diameter, with the effect progressively more pronounced with the order of the resonant mode. This effect shows to some extent in the photographs of the liquid surface (Figures 10 and 11). The motions in each half of the tank during excitation along the long axis appear similar to that during excitation along the short axis. In some of the higher modes there is actually a small step discontinuity in the liquid surface at the joint.

The rotational frequencies (Figure 6) appear to be between the two translational frequencies in the first mode, and somewhat lower than the translational frequencies in the higher modes. Here again, the liquid surface motions appear to be similar to the translational motions along the short axis, but of opposite phase in each half. The step discontinuities in the liquid surface at the joint are quite distinct in many of the cases (Figures 11 and 12).

For translation along the short axis, very little difference in frequencies was noted for a 10° tilt of the tank in either the α or β directions (Figure 5). For translation along the long axis, differences in frequency due to tilt in both directions is measurable. Figure 4 shows the difference increases with the order of the mode, especially at the higher liquid levels.

C. Damping

The plot of logarithmic decrements versus liquid height for excitation along the short (y) axis (Figure 8) indicates generally an initial decrease in damping with increasing liquid height, up to a height-short diameter ratio of about 0.75, and then leveling off to values between about 0.0075 to 0.0125.

A similar plot for excitation along the long (x) axis (Figure 7) shows a leveling off at considerably higher damping than in the other case, approximately 0.020 to 0.035 for the tank tilted in either direction, and approximately 0.04 to 0.05 for the vertical tank. The higher damping along this axis is probably due to the restriction to interchange of liquid between the tank halves offered by the "pinched" portion at the joint.

The rotational damping factors (Figure 9) appear generally to be on the order of magnitude as those due to translation along the short axis.

V. CONCLUSIONS AND RECOMMENDATIONS

One of the more interesting features revealed in this study was the convergence of the higher translational mode frequencies between excitation along the short and long axes of the tank, particularly when the tank was tilted 10° in either direction. For the vertical tank excited along the long axis, the frequencies of the higher modes are significantly higher than with the tank tilted, but little difference was noted for excitation along the short axis.

Little difference (within the data scatter) was noted in damping due to sloshing along the short axis with the tank vertical or tilted 10° in either direction. For sloshing along the long axis, however, the damping in the vertical tank is significantly higher than with the tank tilted 10° in either direction, at the higher modes. The damping due to slosh along the long axis is considerably higher than that due to sloshing along the short axis, for all the tank orientations tested. Damping due to rotational slosh is of the same magnitude as that due to translation along the short axis, at least for the higher liquid levels. Damping, in all cases, appeared to decrease with increasing liquid level, leveling off to a fairly constant value at liquid levels higher than about 0.75 times the short diameter.

The spotty force and moment data indicates the inadequacy of attempting to reduce test time by obtaining only one record of forces and moments at the best obtainable resonant setting of the excitation system. Very small errors, frequency-wise, of this setting can lead to very large differences in the measured forces, moments, and their associated phase relations in a system of such low damping as this. Any future work, on this or other tank configurations, should allow for measurements of forces and moments at several values of frequency in small increments just below and above each resonant peak, to allow plotting the measured data. With damping as low as that in this system, the breakup of the liquid surface prevents accurate, or even repeatable, data to be obtained exactly on resonance.

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